Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8727306241

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

[Turn over

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1 The times taken by members of a large cycling club to complete a cross-country circuit have a normal distribution with mean μ minutes. The times taken, x minutes, are recorded for a random sample of 14 members of the club. The results are summarised as follows, where $\overline{x}$ is the sample mean.

$$\overline{x} = 42.8 \qquad \Sigma(x-\overline{x})^2 = 941.5$$

Find a 95% confidence interval for μ. [4]

2 A large number of students are taking a Physics course. They are assessed by a practical examination and a written examination. The marks out of 100 obtained by a random sample of 15 students in each of the examinations are as follows.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*
Practical examination	66	63	24	52	59	76	88	51	48	36	91	72	68	67	60
Written examination	63	57	39	50	47	71	87	65	56	39	78	70	61	62	70

Use a sign test, at the 10% significance level, to test whether, on average, the practical examination marks are higher than the written examination marks. [5]

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3 A factory produces metal discs. The manager claims that the diameters of these discs have a median of 22.0 mm. The diameters, in mm, of a random sample of 12 discs produced by this factory are as follows.

22.4 20.9 22.8 21.5 23.2 22.9 23.9 21.7 19.8 23.6 22.6 23.0

(a) Carry out a Wilcoxon signed-rank test, at the 10% significance level, to test whether there is any evidence against the manager's claim. [7]

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(b) State an assumption that is necessary for this test to be valid. [1]

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4 The random variable Y is the sum of two independent observations of the random variable X. The probability generating function $G_Y(t)$ of Y is given by

$$G_Y(t) = \frac{t^2}{(4-3t)^4}.$$

(a) Find $E(Y)$. [3]

(b) Write down an expression for the probability generating function of X. [1]

(c) Find $P(X = 4)$. [3]

5 Two companies, P and Q, produce a certain type of paint brush. An independent examiner rates the quality of the brushes produced as poor, satisfactory or good. He takes a random sample of brushes from each company. The examiner's ratings are summarised in the table.

Company	Poor	Satisfactory	Good
P	18	43	64
Q	22	22	31

(a) Test, at the 5% significance level, whether quality of brushes is independent of company. [7]

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(b) Compare the quality of the brushes produced by the two companies. [1]

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6 Jade is a swimming instructor at a sports college. She claims that, as a result of an intensive training course, the mean time taken by students to swim 50 metres has reduced by more than 1 second. She chooses a random sample of 10 students. The times taken, in seconds, before and after the training course are recorded in the table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Time before course	54.2	47.4	52.1	59.0	55.3	51.0	48.9	52.2	58.4	51.4
Time after course	50.1	46.3	52.5	58.8	51.4	48.4	49.5	48.7	58.3	51.4

(a) Test, at the 10% significance level, whether Jade's claim is justified. [7]

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(b) State an assumption that is necessary for this test to be valid. [1]

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7 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{x}{4}(4-x^2) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $\mathrm{Var}\left(\sqrt{X}\right)$. [4]

The continuous random variable Y is defined by $Y = X^2$.

(b) Find the probability density function of Y. [4]

(c) Find the exact value of the median of Y. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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